Hypercube Rhombellanes

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Abstract. Hypercube related rhombellanes are constructed and their mathematical properties detailed. Their ranks or space dimensions, are discussed. Structures are characterized by sequences of connectivity (LC matrix) and rings around vertices (LR matrix), respectively.

Keywords: *polyhedron, n-polytope, hypercube, higher dimensional rhombellane, rank*.

1. Introduction

A regular polyhedron has congruent regular polygons as faces, arranged in the same way around identical vertices; the symmetry group acts transitively on its flags, a regular polyhedron being vertex, edge- and face-transitive [1,2]. There are three symmetry groups: *tetrahedral*; *octahedral* (or cubic) and *icosahedral* (or dodecahedral). Any shapes with icosahedral or octahedral symmetry will also include the tetrahedral symmetry.

There are five regular polyhedra, known as Platonic solids: *tetrahedron* (T), *cube* (C), *octahedron* (O), *dodecahedron* (D) and *icosahedron* (I), written as {3,3}; {4,3}; {3,4}; {5,3} and {3,5}, with the Schläfli [3] symbols {p,q} where p is the number of vertices in a given face while q is the number of faces containing a given vertex. The Platonic solids show pair duals: (cube & octahedron) and (dodecahedron & icosahedron) while the tetrahedron is selfdual. Duality is closely related to *reciprocity* or *polarity*, a geometric operation transforming a convex polyhedron into its dual, this also being a convex polyhedron.

Generalization of a polyhedron to n-dimensions is called a polytope [1,4]. Regular 4-polytopes $\{p,q,r\}$ have cells of the type $\{p,q\}$, faces $\{p\}$, edge figures $\{r\}$ and vertex figures $\{q,r\}$; in words, r-polyhedra (of the type $\{p,q\}$) meet at any edge of the polytope. There are six regular 4-polytopes: 5-cell {3,3,3}; 8-cell {4,3,3}; 16-cell {3,3,4}; 24-cell {3,4,3}; 120-cell {5,3,3} and 600-cell {3,3,5}. Five of them can be associated to the Platonic solids but the sixth, the 24-Cell, has no 3D equivalent. Among them, 5-cell and 24-cell are self-duals while the others are pairs: (8-cell & 16-cell); (120-cell & 600-cell).

The *n-simplex* is a generalization of the triangle or tetrahedron to n-dimensions; it has the Schläfli symbol $\{3^{n-1}\}$ and the number of k-faces $\binom{n+1}{k+1}$. A regular n-simplex can be constructed from a regular $(n-1)$-simplex by connecting a new vertex to all original vertices.

The *hypercube* Q_n is a generalization of the 3-cube to n-dimensions; the Schläfli symbol is $\{4,3^{n-2}\}$ and the number of k-faces is given by $2^{n-k}\binom{n}{k}$. The hypercube can be constructed by the Cartesian product graph of n edges: $(P_2)^{\square n}=Q_n$; the Q_4 hypercube is called 8-cell or also Tesseract.

The *n-orthoplex* or cross-polytope has the Schläfli symbol $\{3^{n-2},4\}$ and k-faces $2^{k+1}\binom{n}{k+1}$; it is the dual of n-cube. The cross-polytope facets are simplexes of the previous dimensions, while its vertex figures are cross-polytopes of lower dimensions.

For general surfaces, Euler [5] characteristic χ can be calculated as an alternating sum of figures of rank k [6-8]:

$$\chi(S) = f_0 - f_1 + f_2 - f_3 + ...,$$

An abstract polytope is a structure that considers only the combinatorial properties of a classical polytope: angles and edge lengths are disregarded. No space, such as Euclidean space, is required to contain an abstract polytope [6-8]. An abstract polytope is a partially ordered set (poset). Every polytope has a dual, of which the partial order is reversed; the dual of a dual is isomorphic to the original. A polytope is self-dual if its dual is the same (i.e., isomorphic to) as the parent. Any abstract polytope may be realized as a geometrical polytope having the same topological structure.

Propellane is an organic molecule consisting of triangle (R_3) rings, a hydrocarbon with formula C_5H_6, first synthesized in 1982 [9]; Its reduced species, C_5H_8, has only square (R_4) rings; it can be represented as $K_{2,3}$ - the complete bipartite graph; it is the smallest rhombellane, rbl.5. The two bridge carbon atoms of the $K_{2,3}$ motif can polymerize, providing a one-dimensional polymer, called staffane [10].

A general procedure, called "rhombellation", to build generalized rhombellanes, was given and illustrated elsewhere [11-13].

A structure is a rhombellane if all the following conditions are obeyed [11,12,14-16]: *(1) All strong rings are squares/rhombs; (2) Vertex classes consist of all non-connected vertices; (3) Omega polynomial has a single term: 1X^|E|; (4) Line graph of the original graph shows a Hamiltonian circuit*; *(5) Structure contains at least one smallest rhombellane rbl.5.*

Omega polynomial $\Omega(x)$ was defined by Diudea (2006) [17,18] on the ground of opposite edge strips *ops* in the graph: $\Omega(x) = \Sigma_s\, m_s X^s$. Its first derivative (in $x = 1$) counts the number of edges "e" in a graph: $\Omega`(1) = \Sigma_s\, sm_s = |E(G) = e|$. There are graphs with a single ops, which is a Hamiltonian circuit of their line-graphs [x]. For such graphs, Omega polynomial has a single term: $\Omega(x) = 1X^e$.

The smallest rhombellane, rbl.5, is the complete bipartite $K_{2.3,}$ graph; any $K_{2.n}$ graph fulfils the five above conditions for rhombellanes. Any $K_{2.n}$ graph contains $n(n\text{-}1)(n\text{-}2)/6$ smallest units rbl.5 = $K_{2.3}$. There are graphs with more than two vertex classes obeying the above conditions, namely those designed by the rhombellation operation.

2. Q_n rhombellanes

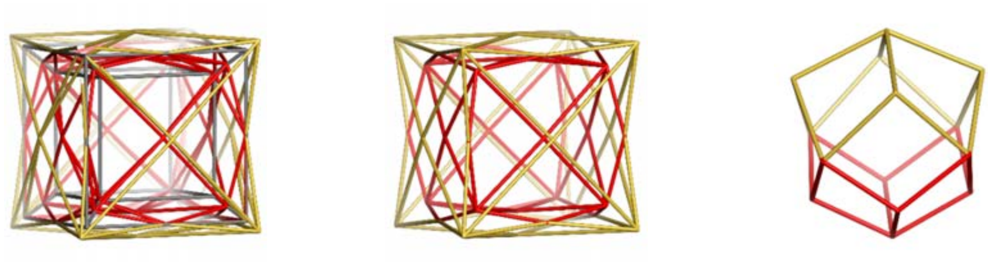
In this paper, a single iteration of rhombellation operation applied on the hypercube Q_n is considered; this results in a single shell (i.e., generation) of Q_n-rhombellanes, *rbl*(Q_n). Figure 1 illustrates this operation on the Tesseract, Q_4 [2]; the number suffixing the name of structures counts their vertices.

Rhombellation adds, at each iteration, a shell consisting of double the number of rhombs in the parent all-rhomb shell (considerations are herein made about the sphere embedded structures). Counting the number of vertices, one can write:

$$v(B_n+1)= v(B_n)+v(Sh_n) = v(B_n)+2(v(B_n)\text{-}1);\ v(Q_{n+1})=2\times v(Q_n);\ B_n=rbl(Q_n);\ sh_n=sh(rbl(Q_n););$$

There is a correspondence between the shell, sh_n, added at each rhombellation operation and the shell of Q_{n+1}, as can be seen in Table 1 (4th and last columns). Difference in edges: $e(rbl(Q_n)$ - $sh(rbl(Q_n)))$ (Table 1, last column) equals the number of rhombs R_4 in Q_{n+1}. The number of rhombs in *rbl*(Q_n) is a function of the number of rhombs $R_4(Q_n)$ and the dimension/rank n: $R_4(\text{rbl}) = R_4(Q_n) \times (n(n + 7)/2 - 6)$. However, *rbl*-operation tends to enlarge the sphere of which the previous shell is embedded while Q_n operation (a graph product by the path p_2) goes in the curled n-space: for each cube-facet,

$Rh_6.8$, by *rbl*-operation, results a rhomb-dodecahedron, $Rh_{12}.14$ (see Fig.1). These operations are not commutative, when applied on a same graph. Other two operations involved in rhomb tessellation, the medial and Poincarė dual: $m_kd=(md)_k$; the medial may be reiterated. Calculation of rank n [2] in Q_4-rhombellane (Table 2) shows $k(rbl(Q_4)) = 5$, one more than the parent Q_4.



rbl(Q_4).56 sh(*rbl*(Q_4).56).40 $Rh_{12}.14$

Figure 1. Rhombellane of Tesseract, Q_4.

Recall that, a vertex figure of an n-polytope is an (n-1)-polytope (e.g. the vertex figure of a 4-polytope is a 3-polytope, or a polyhedron) [2]. A subgraph of an n-polytope, having at least one vertex of deg=n-2, is a tile, t_{n-1}. In the Euler's alternating sum, a tile t_{n-k} is counted as an f_{n-k} facet (see Table 2).

Table 1. Topology of Q_n and their rhombellanes.

n	$v(Q_n)$	$e(Q_n)$	$R_4(Q_n)$	$v(rbl(Q_n))$	$e(rbl(Q_n))$	$R_4(rbl(Q_n))$	v(sh*)	e(sh)	R_4(sh)	$v(rbl(Q_n)$-sh)	$e(rbl(Q_n)$-sh)
3	8	12	6	22	48	54	14	24	12	8	*24*
4	16	32	*24*	56	176	384	40	96	96	16	*80*
5	32	80	*80*	144	560	1920	112	320	480	32	*240*
6	64	192	*240*	368	1632	7920	304	960	1920	64	*672*
7	128	448	*672*	928	4480	28896	800	2688	6720	128	*1792*

*sh= sh(*rbl*(Q_n))

In a previous paper, Diudea [16] defined more strictly the Omega-criterion of rbl-structures: $\Omega(R_{min}.R_{max}) = \Omega(4.4) = 1X^{\wedge}e$. All the shells sh(*rbl*(Q_n)), n = 4,5…are rhombellanes (i.e., obey the five *rbl*-criteria – see the introduction), except for n = 3, for which Ω(sh(*rbl*(Q_3));(4.4)) = 4X^6; even there is a single term for R_{max} = 8, Ω(sh(*rbl*(Q_3));(4.8)) = 1X^e =1X^24, it does not contain rbl.5, thus being neither a rhombellane, *rbl* nor a quasi-rhombellane, *qrbl* structure.

Table 2. Q_4-rhombellane rank calculation

Structure	v	e	R_4	R_6	R_{tot}	rbl.5	Cube	$Rh_{12}.14$	f_3	f_4	X	k
rbl.5	5	6	3	0	3	0	0	0	0	0	2	3
Cube	8	12	6	0	6	0	0	0	0	0	2	3
$Rh_{12}.14$	14	24	12	0	12	0	0	0	0	0	2	3
Q_4	16	32	24	0	24	0	8	0	8	0	0	4
sh(*rbl*(Q_4).56)	40	96	96	0	96	32	0	8	40	0	0	4
rbl(Q_4).56	56	176	384	96	480	344	8	8	360	2	2	5

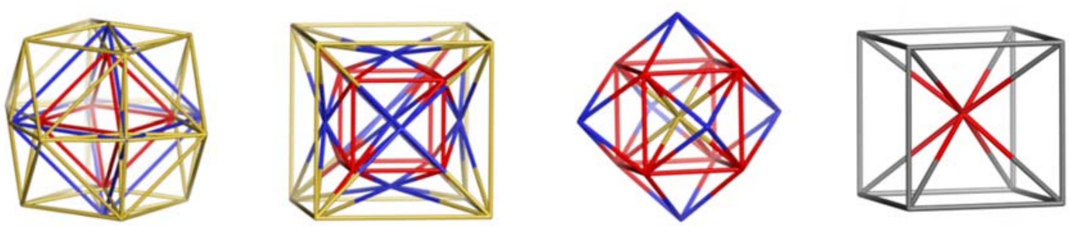
The values returned by Omega polynomial depends on the rings taken into account: R_{min} and R_{max}; these values are put in round brackets. Also, the layer of rings matrix, LR (see below), evidenced different values: one for ($R_{min}.R_{min}$) (that correspond to the ring symbol) and a different one for ($R_{min}.R_{max}$) (see Tables A_4 and A_5).

3. Other high-dimensional cells

The 24-Cell is a convex regular 4-polytope [2], also called "icositetrachoron", "octaplex", or polyoctahedron"; it consists of 24 octahedral cells, with six of them meeting at each vertex and three at each edge; its vertex figure is a cube. The 24-cell is the unique self-dual regular polytope which is neither a polygon nor a simplex; by this reason, it has no analogue in 3D.

The first 8 vertices of 24-cell are the vertices of a regular 16-cell while the remaining 16 are the vertices of the dual 8-cell, or the tesseract Q_4.16. It can be constructed either by *medial* (i.e., *rectification*) of 16-cell, *m*(16-cell), or by *dualization* of 8-cell, *d*(8-cell). There are several 3D projections of 24-cell, of which envelopes are the rhombic dodecahedron Rh_{12}.14, cuboctahedron CO, hexagonal bi-antiprism, elongated hexagonal bipyramid or a tetrakis hexahedron (also named stellated cube *st*(C)).

Keeping in mind the projection of 24-cell with a *st*(C) envelope, a construction of 24-cell as all-body-centered hypercube Q_4.8CP^8.24, (joining eight body-centered CP^8.9 cube units) was proposed by Diudea [11] (Fig. 2).



24-Cell.24 | Q_4.8CP^8.24 | *st*(CP^8).15 | CP^8.9

Figure 2. 24-Cell related structures

4. Hypercube rhombellane relatives

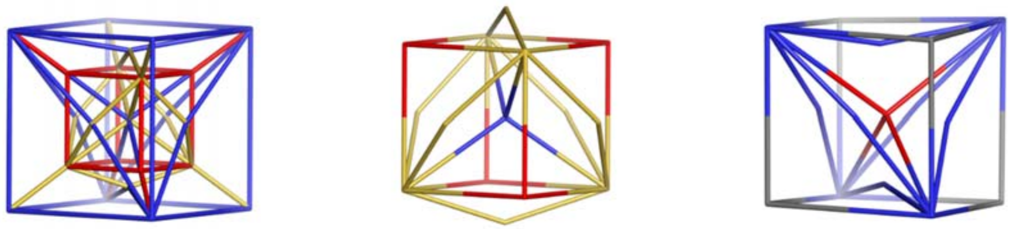
By deleting, in an alternating manner, four edges incident at each central point of CP^8.9 in Q_4.8CP^8.24 (Fig.2) it results in a new structure, Q_4.8CP^4.24(sa) (Fig. 3); the unit is now CP^4.9 (having rbl.5 =10). There are eight CP^4 facets (of rank k = 4) binding Q_4.8CP^4.24(sa); each pair of CP^4 facets shares a facet of rank k = 3, namely the unit rbl.5; thus, Q_4.8CP^4.24(sa) is a 5-polytope (k = 5).

In the figure count (Table 3), two adamantane *ada* units (k = 3) and eight hexagons R_6 (one for each Q_3 face) were considered (see Fig. 3, middle and right); adamantane is a tile, like rbl.5. The total number of rbl.5 = $K_{2.3}$ is 128 = 80 + 4 × 12, the last term coming from $K_{2.4}$. The cluster Q_4.8CP^4.24(sa) [11], above described, is specified as "sa" = (syn.anti), to be differentiated by its isomer "aa" = (anti.anti); this specifications refer to the manner of connecting the two shells of structures to the cube centered points. Figure count data for Q_4.8CP^8.24 and Q_4.8CP^4.24 [11] are given in Table 3.

Table 3. Figure count in Q_4 related structures [11].

Polytope	v	e	R_3	R_4	R_6	2	$K_{2.3}$	$K_{2.4}$	Ada(Py_4)*	3	4	χ	k
CP^8	9	20	12	6	0	18	0	0	(6)	7	-	0	4
Q_4.8CP^8	24	96	96	0	0	96	0	0	0	24	-	0	4
CP^4	9	16	0	18	0	18	10	0	0	11	-	0	4
Q_4.8CP4(sa)	24	64	0	120	8	128	80	12	2	94	8	2	5

* Ada=adamantane tile; Py_4 is the square-based pyramid.



Q_4.8CP^4.24 (sa) Ada.CP^4.15 (inside) Ada.CP^4.15 (outside)

Figure 3. A rhombellanic hypercube Q_4 relative (left); inside (middle) and outside (right) details [11].

Omega polynomial of the two hyper-cuboids consists of a single term, Ω(4.4) = 1X^*e*, saying that all their edges are topologically parallel and thus the structures are rhombellane. The vertex classes have all non-connected points. The 24-Cell, Q_4.8CP^8.24, has Ω (3.3): 96X^1, and its vertex single class have connected vertices; it is not a rhombellane.

Topology of structures herein discussed is basically characterized by sequences of connectivity (LC) and rings around vertex (LR) [19,20]. When all the strong rings are counted, the LR matrix evidenced different values: one for (R_{min}.R_{min}) (that correspond to the ring symbol) and a different one for (R_{min}.R_{max}). Topology of the two isomers of Q_4.8CP^4.24 is given in Table A_5. Computations have been done by the Nano-Studio software program [21].

Conclusions

Generalized rhombellanes are designed by Diudea's rhombellation procedure. Rhombellanes have all the edges topologically parallel, as shown by the single term in Omega polynomial, at the ring-count (4.4) (further involving Hamiltonian circuits visiting their edges). Rhombellanes consist of at least one rbl.5 subgraph. New *n*-polytopes, *n* = 4, 5, related to hypercube Q_4 were proposed and their structures characterized by sequences of connectivity (LC matrix) and rings around vertices (LR matrix).

Rhombellanes represent a new class of structures, with interesting properties, both in theory and applications.

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Appendix

Table A_1. Omega polynomial in hypercube, rhombellane and its shell; ($R_{min}.R_{max}$)

Omega	(4.4)			(4.8)				
n	**Q_n**	**rbl(Q_n)**	**sh(rbl(Q_n))**	**Q_n**	**rbl(Q_n) (rbl.5)**	**sh(rbl(Q_n)) (rbl.5)**	**Type**	**Shells**
3	3x^4	1x^48	*4x^6*	3x^4	1x^48 (22)	*1x^24 (0)*	*non-rbl*	1
4	4x^8	1x^176	1x^96	4x^8	1x^176 (344)	1x^96 (32)	rbl	2
5	5x^16	1x^560	1x^320	5x^16	1x^560 (3120)	1x^320 (320)	rbl	4
6	6x^32	1x^1632	1x^960	6x^32	1x^1632 (20560)	1x^960 (1920)	rbl	8
7	7x^64	1x^4480	1x^2688	7x^64	1x^4480 (110432)	1x^2688 (8960)	rbl	16

Table A_2. Vertex classes in hypercube and its rhombellane (joint substructures-in italics): vertex symbol vs_m; m=1-5 (no. vertices in a class; vertex degree).

n	**$v(Q_n)$**	**vs_1(no; deg)**	**vs_2(no; deg)**	**vs_3(no; deg)**	**vs_4(no; deg)**	**vs_5(no; deg)**
3	22	4^6(4;3)	4^15(4;6)	4^6(4;3)	*4^14(6;6)*	4^6(4;3)
4	56	4^45(8;10)	4^27(8;6)	*4^28(24;6)*	4^24(8;6)	4^12(8;4)
5	144	4^20(16;5)	*4^44(80;6)*	4^105(16;15)	4^75(16;10)	4^60(16;10)
6	368	4^165(32;15)	4^210(32;21)	*4^62(240;6)*	4^120(32;15)	4^30(32;6)
7	928	4^42(64;7)	*4^82(672;6)*	4^315(64;21)	4^378(64;28)	4^210(64;21)

Table A_3. Vertex classes in hypercube rhombellane: connectivity sequence, by LC (see text).

n	**v(Cls)**	**LC_1**	**LC_2**	**LC_3**	**LC_4**	**LC_5**

3	4.4.4.6.4					
		3.10.7.1	3.9.7.2	3.9.9	6.10.4.1	6.8.6.1
4	8.8.24.8.8					
		10.17.22.6	4.24.19.7.1	6.14.25.9.1	6.17.26.6	6.18.18.13
5	16.80.16.					
	16.16	10.20.71.25.15.2	10.26.75.20.11.1	15.26.70.20.11.1	5.50.36.45.7	6.32.36.62.6.1
	32.32.240					
	.					
6	32.32	15.27.166.55.90.13.1	15.37.176.50.81.8	21.37.170.50.81.8	6.50.60.198.30.23	6.90.62.165.27.16.1
	64.672	21.35.337.105.350.49.28.	21.50.357.105.336.35.22.	28.50.350.105.336.35.22.	6.72.90.495.90.167.6.	7.147.99.455.77.133.
7	64.64.64	2	1	1	1	9

Table A4. Vertex classes, sequence of connectivity (LC) and rings around vertex (LR) in sh(*rbl*(Q_n)); ring count at (4.4).

n/*v*(Sh*)	Class (no. vertices; degree)	LC_1	LC_2	LR_1	LR_2
3/14	Cls1: 4^3(8;3) Cls2: 4^4(6;4)	3.6.3.1	4.4.4.1	3.12.18.12.3	4.12.16.12.4
4/40	Cls1: 4^{12}(16;6) Cls2: 4^8(24;4)	6.10.18.5	4.12.12.11	12.48.120.144.60	8.48.96.144.88
5/112	Cls1: 4^{12}(80;4) Cls2: 4^{30}(32;10)	4.24.24.54.4.1	10.15.60.15.10.1	12.120.288.720.648.120.12	30.120.450.720.450.120.30
6/304	Cls1: 4^{16}(240;4) Cls2: 4^{60}(64;15)	4.40.40.178.20.21	15.21.150.35.75.7	16.240.640.2400.2848.1200.336	60.240.1260.2400.2100.1200.420
7/800	Cls1: 4^{20}(672;4) Cls2: 4^{105} (128;21)	4.60.60.455.60. 155.4.1	21.28.315.70.315. 28.21.1	20.420.1200.6300.9100.6300. 3100.420.20	105.420.2940.6300.7350.6300. 2940.420.105

Table A5. Sequence of connectivity (LC) and rings around vertex (LR) in all-centered 8-Cell (Tesseract), Q4.8CP^n.24; *n*=4; 8.

Polytope Rings rbl.5	LC	LR	Ω (R_{min}.R_{min}) (R_{min}.R_{max})	Vertex no. in classes	Degree	Vertex symbol
Q_4.8CP^4.24 (aa)			rbl			
R_4=132	4.8.10.1	12.84.240.180.12 (4.4)	Ω (4.4): 1X^64	2	4	4^12.8^12
R_8=144	4.12.6.1	16.120.232.144.16 (4.4)	Ω (4.8): 1X^64	6	4	4^16.8^40
rbl.5=104	5.12.5.1	21.132.222.132.21 (4.4)		8	5	4^21.8^42
	7.8.7.1	30.132.204.132.30 (4.4)		8	7	4^30.8^69
		24.252.792.588.24 (4.8)				
		56.396.728.444.56 (4.8)				
		63.420.714.420.63 (4.8)				
		99.420.642.420.99 (4.8)				
Q_4.8CP^4.24 (sa)			rbl			
R_4=144	4.14.4.1	18.144.252.144.18 (4.4)	Ω (4.4): 1X^64	16	4	4^18.8^54
R_8=216	8.6.8.1	36.144.216.144.36 (4.4)	Ω (4.8): 1X^64	8	8	4^36.8^108
rbl.5=128		72.576.1008.576.72 (4.8)				
		144.576.864.576.144 (4.8)				
Q_4.8CP^8.24			not rbl			
R_3=96	8.14.1	12.96.168.12 (3.3)	Ω (3.3): 96X^1	24	8	3^12.6^4
R_6=16		16.128.224.16 (3.6)	Ω (3.6): 48X^2	(connected)		
rbl.5=0						